

7004 The Math Part of the Final Exam / Sanghoon Lee

Question 8

(78)

From: \ Into:	Agriculture	Manufacturing
Agriculture:	25 bushels wheat	20 bushels wheat
Manufacturing:	14 yards cloth	6 yards cloth
Labor:	80 years labor	180 years labor
Output:	100 bushels wheat	50 yards of cloth

(i)

$$A = \begin{bmatrix} 1/4 & 2/5 \\ 7/50 & 3/25 \end{bmatrix} \quad l^T = [4/5 \quad 18/5] \quad x^T = [100 \quad 50]$$

$$A \text{ diag}(x) = \begin{bmatrix} 1/4 & 2/5 \\ 7/50 & 3/25 \end{bmatrix} \begin{bmatrix} 100 & 0 \\ 0 & 50 \end{bmatrix} = \begin{bmatrix} 25 & 20 \\ 14 & 6 \end{bmatrix}$$

$$l^T \text{ diag}(x) = [4/5 \quad 18/5] \begin{bmatrix} 100 & 0 \\ 0 & 50 \end{bmatrix} = \begin{bmatrix} 80 \\ 180 \end{bmatrix}$$

(ii) The net product

First way: $100 - (25+20) = 55$ bushels wheat , $50 - (14+6) = 30$ yards cloth

$$\text{Second way : } \begin{bmatrix} 100 \\ 50 \end{bmatrix} - \begin{bmatrix} 1/4 & 2/5 \\ 7/50 & 3/25 \end{bmatrix} \begin{bmatrix} 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 55 \\ 30 \end{bmatrix}$$

(iii) Zero profits

$$(79) p^T = [2 \quad 5] \quad w = 1$$

$$p^T A + w l^T = p^T$$

$$[2 \quad 5] \begin{bmatrix} 1/4 & 2/5 \\ 7/50 & 3/25 \end{bmatrix} + 1 [4/5 \quad 18/5] = [2 \quad 5]$$

(iv) Household sector

(80)

From: \ Into:	Agriculture	Manufacturing	Households	Total
Agriculture:	25 bsh wheat	20 bsh wheat	55 bsh wheat	100 bsh wheat
Manufacturing:	14 yd cloth	6 yd cloth	30 yd cloth	50 yd cloth
Households:	80 yr labor	180 yr labor	40 yr labor	300 yr labor
Output:	100 bsh wheat	50 yd cloth	300 yr labor	

$$A = \begin{bmatrix} 1/4 & 2/5 & 11/60 \\ 7/50 & 3/25 & 1/10 \\ 4/5 & 18/5 & 2/15 \end{bmatrix} \quad x^T = [100 \quad 50 \quad 300] \quad p^T = [2 \quad 5 \quad 1]$$

$$Ax = x :$$

$$\begin{bmatrix} 1/4 & 2/5 & 11/60 \\ 7/50 & 3/25 & 1/10 \\ 4/5 & 18/5 & 2/15 \end{bmatrix} \begin{bmatrix} 100 \\ 50 \\ 300 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \\ 300 \end{bmatrix}$$

$$p^T A = p^T :$$

$$\begin{bmatrix} 2 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 2/5 & 11/60 \\ 7/50 & 3/25 & 1/10 \\ 4/5 & 18/5 & 2/15 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 1 \end{bmatrix}$$

(v) A diag(x):

$$\begin{bmatrix} 1/4 & 2/5 & 11/60 \\ 7/50 & 3/25 & 1/10 \\ 4/5 & 18/5 & 2/15 \end{bmatrix} \begin{bmatrix} 100 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 300 \end{bmatrix} = \begin{bmatrix} 25 & 20 & 55 \\ 14 & 6 & 30 \\ 80 & 180 & 40 \end{bmatrix}$$

(vi)

(81)

From: \ Into:	Agriculture	Manufacturing	Households	Total
Agriculture:	\$ 50 wheat	\$ 40 wheat	\$ 110 wheat	\$ 200 wheat
Manufacturing:	\$ 70 cloth	\$ 30 cloth	\$ 150 cloth	\$ 250 cloth
Households:	\$ 80 labor	\$ 180 labor	\$ 40 labor	\$ 300 labor
Output:	\$ 200 wheat	\$ 250 cloth	\$ 300 labor	

Manufacturing pay more for agricultural inputs than manufacturing inputs.
About 27% of total household income is spent on agricultural goods.

(vii)

diag(p) A diag(x):

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 2/5 & 11/60 \\ 7/50 & 3/25 & 1/10 \\ 4/5 & 18/5 & 2/15 \end{bmatrix} \begin{bmatrix} 100 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 300 \end{bmatrix} = \begin{bmatrix} 50 & 40 & 110 \\ 70 & 30 & 150 \\ 80 & 180 & 40 \end{bmatrix}$$

Question 9

If $A \geq 0$ is indecomposable, $l \geq 0$, $w \geq 0$, $r \geq -1$ and $p \geq 0$, then

(84) $(1+r)p^T A + wl^T = p^T$ can only hold if $r \leq r^*$.

$$(1+r^*)p^T A = p^T \Rightarrow (1+r)p^T A + wl^T = (1+r^*)p^T A \Rightarrow r \leq r^*.$$

or

Using (55) if $q^T A \leq \mu q^T$ ($q \geq 0$) then $\mu \geq \lambda$

$$p^T A + wl^T = [1/(1+r)]p^T \Rightarrow [1/(1+r)] \geq [1/(1+r^*)] \Rightarrow r \leq r^*.$$

Question 21

The non-substitution theorem says that the choice of technique is independent of patterns

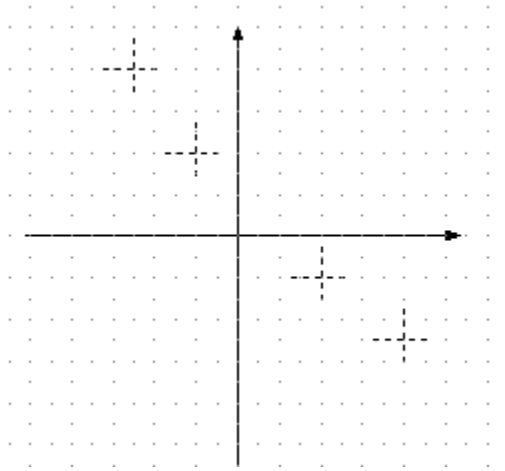
of final demand when efficiency prevails as to the use of a single primary factor, say labor, and that the commodity price vector is uniquely determined by the production structure, independently of final demand patterns.

Question 22

(202)

$$A = \begin{bmatrix} 0 & 0 & 2/3 & 1/2 \\ 2/3 & 1/2 & 0 & 0 \end{bmatrix} \quad I^T = [1 \ 2 \ 1 \ 2] \quad B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(i)



(ii)

Among lines, the line between the first process and the third process shows the most largeness in quantity.

Question 23

From: \ Into:	Department II	Department I
Department I:	1500 tons iron	4000 tons iron
Labor:	1500 years labor	2000 years labor
Output:	3000 bushels wheat	6000 tons iron

(i)

$$A = \begin{bmatrix} 0 & 0 \\ 1/2 & 2/3 \end{bmatrix} \quad I^T = [1/2 \ 1/3] \quad x^T = \begin{bmatrix} 3000 \\ 6000 \end{bmatrix}$$

(ii)

$$(I - A)^{-1} = \begin{bmatrix} 1 & 0 \\ -1/2 & 1/3 \end{bmatrix}^{-1} = 3 \begin{bmatrix} 1/3 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/2 & 3 \end{bmatrix}$$

(iii)

$$\lambda^T = 1^T(I - A)^{-1} = [1/2 \quad 1/3] \begin{bmatrix} 1 & 0 \\ 3/2 & 3 \end{bmatrix} = [1 \quad 1]$$

(vi)

Department II invests 150 out of its surplus-value of 750, i.e., 20%.

(viii)

(213)

Reproduction Scheme I	Department II	Department I
Constant capital:	\$ 1500	\$ 4000
Variable capital:	\$ 750	\$ 1000
Surplus value:	\$ 750	\$ 1000
Total value:	\$ 3000	\$ 6000

Total surplus value is 1750, and total investment is 659.
 $a + b = 659$. $4/5 a + 2/3 b = 500$. $a = 455$, $b = 204$

(220)

Reproduction scheme 2a	Department II	Department I
Constant capital:	\$ 1638	\$ 4364
Variable capital:	\$ 816	\$ 1091
Surplus value:	\$ 816	\$ 1091
Total value:	\$ 3270	\$ 6546

Question 33

$$\dot{L} / L = \rho = 0.02$$

$$Q = K^\alpha L^{1-\alpha} = K^{1/2} L^{1/2} = (KL)^{1/2}$$

$$q = (K/L)^{1/2} = k^{1/2} = f(k)$$

$$k' = sf(k) - \rho k = sk^{1/2} - 0.02k$$

$$\text{max consumption: } f(k) - \rho k = k^{1/2} - 0.02k$$

$$\Rightarrow 1/2 k^{-1/2} - 0.02 = 0 \quad \Rightarrow \quad k = 625$$

$$K / L = k = 625 \quad \Rightarrow \quad r = f(k) = 1/2 k^{-1/2} = 0.02$$