Answer to Problem 6

Consider firm 1. $MR_1 = (a - bq_2) - 2bq_1$

$MR = MC$

$$q_1 = \frac{1}{2}\left(\frac{a-c_1}{b} - q_2\right) = \frac{1}{2}(s_1 - q_2), \quad MC_1 = c_1, \quad s_1 = \frac{a-c_1}{b}$$

By the same logic:

$$q_2 = \frac{1}{2}(s_2 - q_1) \text{ and } s_2 = \frac{a-c_2}{b}$$

Then $q_1 = \frac{1}{2}(s_1 - q_2) = \frac{1}{2}\left(s_1 - \frac{1}{2}(s_2 - q_1)\right) = \frac{1}{2}s_1 - \frac{1}{4}s_2 + \frac{1}{4}q_1$

$$\frac{3}{4}q_1 = \frac{1}{2}s_1 - \frac{1}{4}s_1 + \frac{1}{4}s_1 - \frac{1}{4}s_2$$

$$q_1 = \frac{1}{3}s_1 + \frac{1}{3}(s_1 - s_2)$$

$$= \frac{1}{3}s_1 + \frac{1}{3}\left(\frac{c_2 - c_1}{b}\right) \quad \text{[result 1]}$$

$\frac{2}{4} \times \frac{1}{1}$

$\frac{2}{3}s_1 - \frac{1}{3}s_1 + \frac{1}{3}s_1 - \frac{1}{3}s_2$

Next,

$$q_2 = \frac{1}{2}(s_2 - q_1) = \frac{1}{2}\left(s_2 - \left(\frac{1}{3}s_1 + \frac{1}{3}(s_1 + s_2)\right)\right)$$

$$q_2 = \frac{1}{2}\left(\frac{4}{3}s_2 - \frac{2}{3}s_1\right) = \frac{2}{3}s_2 - \frac{1}{3}s_1$$

$$= \frac{2}{3}s_2 - \frac{1}{3}s_1 - \frac{1}{3}s_1 + \frac{1}{3}s_1$$

$$= \frac{1}{3}s_1 - \frac{2}{3}(s_1 - s_2) = \frac{1}{3}s_1 - \frac{2}{3}\left(\frac{a-c_1}{b} - \frac{a-c_2}{b}\right)$$

$$= \frac{1}{3}s_1 - \frac{2}{3}\left(\frac{c_2 - c_1}{b}\right)$$

Answer to Problem 6 Cont

Part Three!

$$p = a - b(q_1 + q_2)$$

$$= a - b\left(\frac{1}{3}S_1 + \frac{1}{3}\left(\frac{c_2 - c_1}{b}\right) + \frac{1}{3}S_1 - \frac{2}{3}\left(\frac{c_2 - c_1}{b}\right)\right)$$

$$= a - b\,\frac{2}{3}S_1 - b\left(-\frac{1}{3}\left(\frac{c_2 - c_1}{b}\right)\right) =$$

$$= a - \frac{2}{3}bS_1 - \frac{1}{3}bS_1 + \frac{1}{3}bS_1 + \frac{1}{3}(c_2 - c_1)$$

$$= a - bS_1 + \frac{1}{3}bS_1 + \frac{1}{3}(c_2 - c_1)$$

$$= a - b\left(\frac{a - c_1}{b}\right) + \frac{1}{3}bS_1 + \frac{1}{3}(c_2 - c_1)$$

$$= c_1 + \frac{1}{3}bS_1 + \frac{1}{3}(c_2 - c_1)$$