Game Theory is a mathematically based methodology for analyzing *rational decision making under conditions of interdependent choices.*

Elements of Game Theory:
- Players
- Strategies
- Payoffs

Games are divided into:
a. Non-cooperative / Cooperative
b. Static (or Simultaneous) / Dynamic (or Sequential)
c. One Shot / Repeated

1 Static Games

1.1 Static Games and Normal Form (or Strategic Form)

Prisoners' Dilemma Two individuals, Steve and Jan, are arrested for a crime they have committed together. They are placed in separate rooms for questioning and are not allowed to communicate with one another. Each prisoner can confess to the crime, thereby implicating the other person, or deny involvement. If only one prisoner confesses, that person will serve six months in prison, while the other will receive the maximum sentence of 20 years. If both confess, each will serve 10-year terms; however, if neither confesses, each will serve one year.

Normal Form (or Strategic Form):

		Jan	
		Deny	Confess
Steve	Deny	$-1, -1$	$-20, -0.5$
	Confess	$-0.5, -20$	$-10, -10$

Battle of the Sexes At the separate workplaces, Chris and Pat must choose to attend either an opera or a prize fight in the evening. Both would like to spend the evening together. Chris prefers the opera and Pat prefers the prize fight.

Matching Pennies Each of the two players has a penny. Two players choose whether to show the head or the tail. If two pennies match (both heads or both tails) then players 2 wins player 1's penny. Otherwise, player 1 wins player 2's penny.

1.2 Nash Equilibrium

Nash Equilibrium is an outcome where each player selects the strategy that optimizes its payoff, given the strategies chosen by the other player.

In Nash equilibrium, no one has an incentive to change his/her strategy given the strategy choices of the others.

In Prisoners' Dilemma, (Confess, Confess) is Nash equilibrium. This result shows that individually rational action results in both persons being made worse off in terms of their own self-interested purposes.

How to Find Nash Equilibrium

i. Cell-by-Cell Inspection

ii. Best Response Function: In a 2-player game, (s_1, s_2) is a Nash equilibrium if and only if player 1's strategy s_1 is her/his best response to player 2's strategy s_2, and player 2's strategy s_2 is her/his best response to player 1's strategy s_1.

iii. Iterated Elimination of Strictly Dominated Strategies:

Suppose s_i and s_i' are two strategies for players i. s_i' is *strictly dominated* by s_i if i's payoff from s_i is strictly greater than i's payoff from s_i' regardless of other players' choices. (s_i' is *weakly dominated* by s_i if i's payoff from s_i is at least as great as i's payoff from s_i' regardless of other players' choices.)

A rational player never chooses a strictly dominated strategy. Thus, any strictly dominated strategy can be eliminated. If we can eliminate all but the strategies s^*, then s^* is the unique Nash equilibrium.

	N	L	J
N	73,25	57,42	66,32
C	80,26	35,12	32,54
J	28,27	63,31	54,29

1.3 More Games

Cartel Two producers agree to produce the combined level of output that maximizes their joint profit. Each firm faces two strategies: abide by its production agreement or cheat on it.

		B	
		Not Cheating	Cheating
A	Not Cheating	\$15,000, \$15,000	\$5000, \$20,000
	Cheating	\$20,000, \$5000	\$10,000, \$10,000

Big Monkey and Little Monkey Big Monkey and Little Monkey eat warifruit. To get the warifruit, at least one of the monkeys must climb the tree and shake vigorously until the fruit falls to the ground. A warifruit is worth 10Kc (kilocalories) of energy. Running up the tree, shaking the fruit loose, and running back down to the ground costs 2Kc for Big Monkey, but negligible for Little Monkey. If both monkeys climb the tree, Big Monkey gets 7Kc and Little Monkey gets 3Kc. If only Big Monkey climbs the tree, Big Monkey gets 6Kc and Little Monkey gets 4Kc. If only Little Monkey climbs the tree, Big Monkey gets 9Kc and Little Monkey gets 1Kc.

Tourists and Natives There are only two bars in a city. They can charge price of \$2, \$4 or \$5. 6,000 tourists pick a bar randomly. 4,000 natives select the lowest price bar.

Battle of the Networks I

Two television networks are battling for shares of viewers. Network 1 has an advantage in sitcoms. If it runs a sitcom, it always gets a higher share than if it runs a game show. Network 2 has an advantage in game shows. If it runs a game show, it always gets a higher share than if it runs a sitcom.

		Network 2: Sitcom	Network 2: Game Show
Network 1	Sitcom	55%, 45%	52%, 48%
	Game Show	50%, 50%	45%, 55%

Battle of the Networks II: There are three television networks.

N3 chooses Sitcom

		N2: S	N2: G
N1	S	38%, 31%, 33%	33%, 35%, 32%
	G	32%, 34%, 34%	30%, 36%, 34%

N3 chooses Game Show

		N2: S	N2: G
N1	S	36%, 30%, 34%	35%, 35%, 30%
	G	33%, 34%, 33%	33%, 35%, 32%

Price-Matching as Tacit Collusion I Bernie and Manny both sell VCRs, and both have per-unit costs of \$250. They compete on price: the low price seller gets all the market, and they split the market if they have equal prices.

Price-Matching as Tacit Collusion II Suppose the monopoly price is \$300. Bernie advertises that if a customer buys a VCR from him for \$300 and discovers he or she can buy it cheaper at Manny's, Bernie will sell the customer the VCR with a rebate equal to twice the price difference between the two stores. Manny does the same thing.

Duopoly: $P = 10 - Q, \quad C = 0.$
Perfect Competition: $\pi^* = \$0$.
Cournot: $\pi^* = \$22.28\,(\pi_1^* = \$11.09, \pi_2^* = \$11.09)$.
Stackelberg: $\pi^* = \$18.75\,(\pi_1^* = \$12.50, \pi_2^* = \$6.25)$

	Act as Follower	Act as Leader
Act as Follower	\$11.09, \$11.09	\$6.25, \$12.50
Act as Leader	\$12.50, \$6.25	\$0, \$0

Airlines Consider the case of two airlines, Delta and American, each offering a daily flight from Boston to Budapest. Assume that each firm has already set a price for the flight, but that the departure time is still undecided. Suppose that 60% of the potential clientele would prefer to leave Boston in the evening. The remaining 40% prefer a morning Boston departure. Assume that whenever the two carriers schedule identical departure times, Delta gets 60% of the market and American gets only 40%.