- Be sure to show how you got the answer, i.e., do not just show the answer; go through the steps.
- Due: July 15th, 9:00am

1. In a perfectly competitive market, a production function is $Q = \frac{1}{2}K^{.5}L^{.5}$. $P_K = \$2$ and $P_L = \$8$.
a. Find the firm's marginal product of labor function (MP_L). Does the MP_L exhibit diminishing marginal returns? Explain.
b. Derive the function for the marginal rate of technical substitution. Does the $MRTS_{LK}$ diminish as labor increases? Explain.
c. What type of returns to scale are exhibited in the production function? Explain.
d. Find the long-run cost function by using the Lagrangian method.
e. Assume that, in the short run, the firm's capital is fixed at 4 units. Find the short-run total cost function, the marginal cost function and the average variable cost function.
f. Assume that the price of the product sold by the firm is \$32. Determine the firm's short-run profit-maximizing level of output of the good (Q^*) and the amount of profit (or loss) earned (π^*).
g. Draw the short-run supply curve.

2. The short-run total cost function for a perfectly competitive firm of good X is $C(Q) = 2+28Q-3Q^2+\frac{1}{3}Q^3$. Draw the short-run supply curve for this firm.

3. Suppose a firm in a perfectly competitive market has a total cost function of $C = \frac{1}{3}Q^3 - 2Q^2 + 16Q + 100$ and the market price is \$12.
a. Find the firm's marginal cost function and the average variable cost function.
b. Find the firm's optimal (i.e., profit-maximizing) output (Q^*) and profit (π^*).
c. Draw the short-run supply curve. Will the firm produce goods? Explain.

4. Sarah's pretzel plant has the following short-run cost function:
$C = \frac{wQ^3}{1000K^{3/2}} + 50K$, where Q is Sarah's output level, w is the cost of a labor hour, and K is the number of pretzel machines Sarah leases. At the moment, Sarah leases 10 pretzel machines, the cost of a labor hour is \$6.85, and she can sell all the output she produces at \$35 per unit.
a. Find Sarah's optimal output (Q^*) and profit.
b. Now suppose that the cost per labor hour rises to \$7.50. Find Sarah's optimal output (Q^*) and profit (π^*).

5. Homer's boat manufacturing cost function is: $C = \frac{75}{128}Q^4 + 10240$. Homer can sell all the boats he produces for \$1200.
a. Find his optimal output (Q^*) and profit (π^*).
b. Will he produce boats? Explain.

End