

There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are almost equally weighted (they are worth 13 points, 13 points, and 10 points), while Prof. Kiefer's required question is worth 18 points, which is twice as much as his optional questions.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 13 points; the second is worth 13 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 10 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It is helpful (but not required) if you put the number of the problem you are working on at the top of every page.

Good luck.

Section 1.

Answer all of the following three questions.

1. **[13 points]** Suppose a price-taking consumer consumes two commodities x and y and has an indirect utility function of the form

$$v(\bar{p}_x, \bar{p}_y, m) = \ln \left[\frac{\alpha^\alpha \beta^\beta m^{\alpha+\beta}}{\bar{p}_x^\alpha \bar{p}_y^\beta (\alpha + \beta)^{\alpha+\beta}} \right]$$

where m is the consumer's income, \bar{p}_x is the price of x , and \bar{p}_y is the price of y .

- (a) Show that this consumer's expenditure function is

$$e(\bar{p}_x, \bar{p}_y, u) = (\alpha + \beta) \left(\frac{\bar{p}_x^\alpha \bar{p}_y^\beta}{\alpha^\alpha \beta^\beta} \right)^{\frac{1}{\alpha+\beta}} \exp \left(\frac{u}{\alpha + \beta} \right)$$

where exponentiation is denoted by “exp” to avoid confusion with the notation for the expenditure function e .

- (b) Form this consumer's money metric indirect utility function,

$$\mu(\hat{p}_x, \hat{p}_y; \bar{p}_x, \bar{p}_y, m) \equiv e(\hat{p}_x, \hat{p}_y, v(\bar{p}_x, \bar{p}_y, m)).$$

Your final expression should not explicitly involve the indirect utility function v .

- (c) Show that this consumer's utility function is either

$$u(x, y) = \alpha \ln x + \beta \ln y$$

or a monotonically increasing function of this (such as $x^\alpha y^\beta$).

2. **[13 points]** On August 22, 2010, the *Los Angeles Times* published an opinion piece entitled “Disincentivizing Greed” written by Neal Grabler (who was then a public policy scholar at the Woodrow Wilson Center in Washington, DC). Here is an excerpt of the piece; it essentially argues that decreasing tax rates increases the amount of dishonest labor, which is an assertion about a comparative statics derivative.

To a surprising degree, economic misfortune has correlated with low top marginal tax rates. The top marginal tax rate at the time of the 1929 crash was 24%. After his election, Roosevelt promptly raised it to 63% and then to 94%, and one could easily make the case that it was this rise, rather than financial regulation, that played the primary...role in curbing abuses by attacking greed at its source, without, by the way, damaging the economy. Roosevelt essentially taxed away big money.

During the long postwar economic boom, the top marginal rates hovered at 91%, removing a lot of the incentive to game the financial system. There was no point in scheming if you couldn't profit from it. Still, the country prospered. So did Wall Street.

Then came the greed deluge...[W]hen President Reagan cut the top marginal tax rate drastically from 70% to 50% in 1981 and then to 28% in 1988 (putting aside for the moment the cut in the capital gains tax and other investment incentives), that's when the troubles began—from the S&L crisis right through to the fall of Lehman Bros. It wasn't enough for the rich to be rich. Human nature being what it is, they had to be super-rich. Or put another way, tax cuts, including the Bush tax cuts, fed some of the worst aspects of human nature and led to some of the worst excesses. It was just a matter of time before Wall Street went wild.

When the fire of greed is stoked this way, financial reforms cannot possibly bank it...We now live in a country that seems to worship wealth, and we may just have to live with the consequences—a Bernie Madoff, an Enron, a Lehman Bros., and a steep recession when the super-rich overplay their hand. The alternative is regulation that goes to the source by raising those marginal tax rates (and capital gains taxes) and forcing the super-rich to merely be rich again...

- (a) Argue that a reasonable way—certainly not the only way, but a reasonable way—to model the (indirect) utility that the “rich” or “super-rich” people described in this article get from their pretax income is

$$\text{“honest income”} + \sqrt{\text{“dishonest income”}} .$$

(This is not a standard way of modeling indirect utility, of course.)

(b) If the tax rate is t , interpret

$$\begin{aligned} & \text{“honest income”} + \sqrt{\text{“dishonest income”}} \\ & - t \cdot (\text{“honest income”} + \text{“dishonest income”}). \end{aligned}$$

(c) Modelling “income” as a wage rate (consider an “honest wage” and a “dishonest wage”) times a number of hours worked (consider “honest labor time” and “dishonest labor time”) and imposing some constraint on the number of hours a human can work, discuss whether or not the expression in part (b) supports Grabler’s hypothesis by calculating an appropriate comparative statics derivative. Does the appropriate second-order condition hold?

Hint: If you substitute the constraint on working hours into the objective function, the new problem has only one endogenous variable, which is much easier to work with.

3. [18 points]

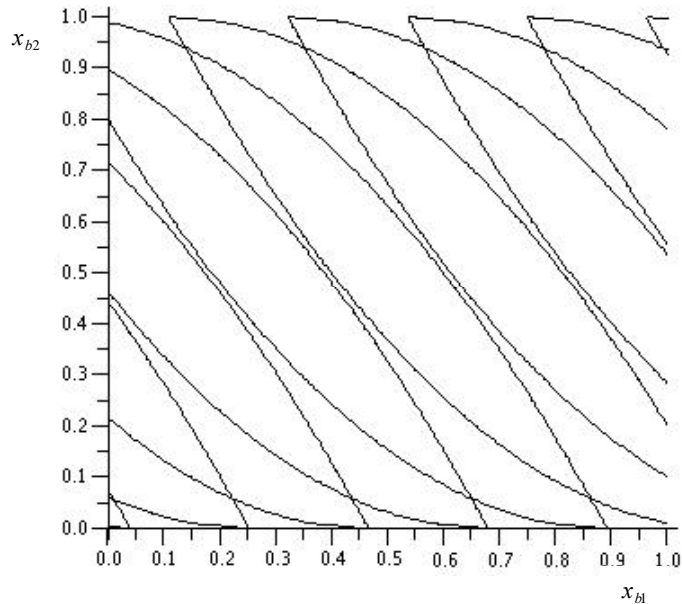
Story One: Barbie and Ken consume two private goods, coffee x_1 , and croissants x_2 . The utility functions and endowments are given as follows:

$$\begin{array}{ll} \text{Barbie} & U_b = x_{b1} + \sqrt{x_{b2}} \quad \omega_b = (0.35, 0.95), \\ \text{Ken} & U_k = x_{k1} + \sqrt{x_{k2}} \quad \omega_k = (0.65, 0.05). \end{array}$$

A feasible general equilibrium is described by $0 = \omega_{b1} + \omega_{k1} - x_{b1} - x_{k1}$ and $0 = \omega_{b2} + \omega_{k2} - x_{b2} - x_{k2}$. Barbie and Ken agree on the social welfare function,

$$W = \min(U_b, U_k).$$

The diagram below plots the indifference curves of both with respect to Barbie's consumption bundle.



- Identify the endowment point and the core. Find the Walrasian equilibrium from the given endowment. What is the equilibrium price vector? Illustrate your answer.
- Find the Pareto set (contract curve) and the social optimum. Illustrate your answer.

Story Two: Barbie and Ken consume a private good, coffee x_i , and a public good, poetry G . The utility functions and endowments (of coffee) are given as follows:

$$\begin{array}{ll} \text{Barbie} & U_b = x_b + \sqrt{G} \quad \omega_b = 1, \\ \text{Ken} & U_k = x_k + \sqrt{G} \quad \omega_k = 1. \end{array}$$

Both may make a contribution g_i toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i.$$

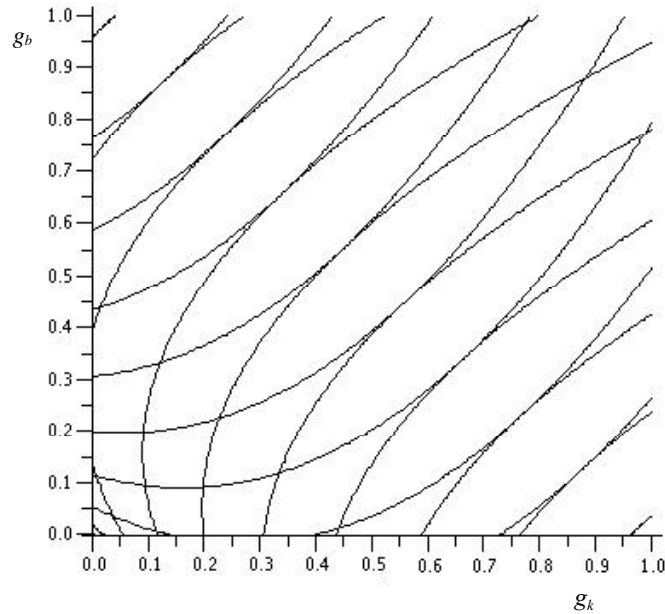
The coffee can be transformed into poetry according to the transformation function

$$0 = x_b + x_k + g_b + g_k - \omega_b - \omega_k.$$

Again, Barbie and Ken agree on the social welfare function,

$$W = \min(U_b, U_k).$$

The diagram below plots the indifference curves of both in contribution space.



- (c) Show that there are multiple Nash equilibriums. Find the Lindahl equilibrium. Illustrate your answer. Would both always favor a move from a Nash to the Lindahl?
- (d) Find the Pareto set and the social optimum.
- (e) How are the two stories similar? How do they differ? Discuss how the First Theorem of Welfare Economics applies to each.
- (f) Could an allocation fail to be Pareto efficient, but still be socially optimal? Could it fail to be socially optimal, but still be Pareto efficient? Does your answer differ between the two stories?

Section 2.

Answer one of the following two questions.

1. [10 points] Suppose a competitive firm has a production possibilities set denoted Y , an input requirement set denoted $V(y)$, and a production function denoted $f(\mathbf{x})$.
 - (a) Prove that if Y is a convex set then $V(y)$ is a convex set.
 - (b) Prove that the converse of the statement in part (a) is false; this is easily done by making a graph of a counterexample.
 - (c) Prove that $V(y)$ is a convex set if and only if $f(\mathbf{x})$ is a quasiconcave function.
2. [10 points] Suppose a competitive firm has a profit function denoted $\pi(\mathbf{p})$ and a production possibilities set denoted by Y whose generic element is denoted by \mathbf{y} .

In solving the problems below, if you use Hotelling's Lemma, you should prove it (using the Envelope Theorem).

- (a) Show that $\pi(\mathbf{p})$ is increasing in output prices and decreasing in input prices.
- (b) Show that $\pi(\mathbf{p})$ is homogeneous of degree one in \mathbf{p} .
- (c) Show that $\mathbf{y}^*(\mathbf{p})$ is homogeneous of degree zero in \mathbf{p} .
- (d) Varian (p. 41) writes that properties such as these (emphasis added by me):

... follow from the definition of the profit function alone
and *do not rely on any properties of the technology*.

Why do such properties actually depend on technology, in the sense that there are some technologies for which $\pi(\mathbf{p})$ is not even defined for a competitive firm?

Section 3.

Answer two of the following three questions.

1. [9 points]

Imagine a duopoly game with the following profit payoffs.

profit payoffs (Microsoft, Apple)		Apple	
		passive	aggressive
Microsoft	passive	(8,3)	(4,4)
	aggressive	(9,0)	(2,-3)

Think of this as a nonspecific game, not necessarily Cournot or Bertrand. Only two strategies are available.

- (a) Consider a single simultaneous game. Does either player have a dominant strategy? Is there more than one Nash equilibrium?
- (b) Now consider an infinite number of repetitions of the simultaneous game. Are there any conditions under which the (passive, passive) outcome is a Nash equilibrium? If (passive, passive) is an equilibrium, is it subgame-perfect?
- (c) Suppose that Microsoft moves first, and that only one game is played. Draw the extensive form of this sequential game. What is the subgame-perfect Nash equilibrium? Discuss.

2. [9 points]

Imagine a 2 by 2 economy with two consumers, $i = 1$ (Robinson) and 2 (Friday); each consumes two goods, leisure x_{1i} and fish x_{2i} . Their preferences are identical,

$$u(x_{1i}, x_{2i}) = x_{1i} - \frac{(x_{2i} - 3)^2}{2}.$$

Their endowments $\omega = (\omega_{1i}, \omega_{2i}) = (3, 0)$ are also identical. Robinson owns a fish firm. Fish can be produced according to the production function $y_2 = |y_1|$. General equilibrium is described by $x_{1i} = \omega_{1i} + y_{1i}$, $y_{11} + y_{12} = y_1$ and $x_{21} + x_{22} = y_2$. Define the price of leisure as 1.

- (a) In perfect competition what is equilibrium price of a fish and the allocation $(x_{11}, x_{21}, x_{12}, x_{22})$? Show that profits are zero.
- (b) Consider a pure monopoly regime for the fish market; remember that Robinson alone owns the fish firm. Now what are profits? What is equilibrium price of a fish and the allocation?
- (c) Consider a reform of the Robinson-monopoly regime in favor of perfect competition. Is this a Pareto improvement? Discuss the wider implications of this example.

3. [9 points]

A democratic society consists of many citizens, identical except for their employment status. There are only two time periods: the present ($t = 1$) and the future ($t = 2$). Each individual has the following utility function,

$$U^j = E(\ln(c_1^j) + \ln(c_2^j)), \quad j \in \{\text{employed, unemployed}\}$$

where e denotes being employed, u denotes being unemployed and c_t^j is consumption in the t^{th} period.

The unemployment rate in period 1 is $u_1 = 0.10$. The probability that an employed in period 1 will lose her job for period 2 is $\phi = 0.04$ (the firing rate), while the probability that an unemployed will gain a job is $v = 0.36$ (the hiring rate).

During the first period an election sets a tax τ on the employed during the second period to finance the unemployment insurance benefit f . Total tax collections equal benefits paid. In the first period employed consumption is $c_1^e = 1$, and unemployed consumption is $c_1^u = 0$; in the second period $c_2^e = 1 - \tau$ and $c_2^u = f$. On election day voters know their employment status in period 1, but not in period 2.

- (a) What tax does the employed majority prefer? What is the implied benefit level?
- (b) A social planner has a Benthamite welfare function defined on the allocation in period 2. What tax and benefit level would this planner prefer?
- (c) Explain why the adverse selection and moral hazard issues are not relevant in this example. Discuss the wider implications of this model for studying social conflict.